Exercise 33

Use the Intermediate Value Theorem to show that there is a root of the equation in the given interval.

$$x^5 - x^3 + 3x - 5 = 0, \quad (1, 2)$$

Solution

Let $f(x) = x^5 - x^3 + 3x - 5$. This is a polynomial function, so it's continuous everywhere on the closed interval $[1, 2]$. Find a value of x in this interval for which $f(x)$ is negative, and find a value of x in this interval for which $f(x)$ is positive.

$$f(1) = (1)^5 - (1)^3 + 3(1) - 5 = -1$$

$$f(2) = (2)^5 - (2)^3 + 3(2) - 5 = 25$$

$N = 0$ lies between $f(1)$ and $f(2)$, so by the Intermediate Value Theorem, there exists a root in the open interval $(1, 2)$.